Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

BTECH I Year I Semester Supplementary Examinations November-2020

ALGEBRA AND CALCULUS

(Common to all branches)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1 Find the characteristic equation of $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$ and hence compute A^{-1}. Also find the matrix representation of $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. **12M**

OR

2 **a** Show that the matrix $A=\begin{bmatrix}1&-2&2\\1&2&3\\0&-1&2\end{bmatrix}$ satisfies its characteristic equation and find A^{-1} ? **8M**

b Reduce the matrix A=$\begin{bmatrix}-2&-1&-3&-1\\1&2&3&-1\\1&0&1&1\\0&1&1&-1\end{bmatrix}$ into Echelon form and find its rank? **4M**

UNIT-II

3 **a** Verify Cauchy's mean value theorem for $f(x)=\sin x$ and $g(x)=\cos x$ in $\left[0,\frac{\pi}{2}\right]$. **6M**

b Obtain the Maclaurin's series expression of the following function of $\cos x$ **6M**

OR

4 **a** Verify Rolle's theorem for $f(x)=\log\left(\frac{x^2+6}{5x}\right)$ in $(2,3)$. **6M**

b Show that $1+x+\frac{x^2}{2}\le e^x\le 1+x+\frac{x^2}{2}e^x$ for every $x\ge 0$. **6M**

UNIT-III

5 **a** If $f(x,y)=\frac{1}{\sqrt{y}}e^{\frac{-(x-a)^2}{4y}}$, then prove that $f_{xy}=f_{yx}$. **6M**

b Calculate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ if $u=\frac{x}{\sqrt{1-t^2}}, v=\frac{y}{\sqrt{1-t^2}}$ and $w=\frac{z}{\sqrt{1-t^2}}$ where $t=\sqrt{x^2+y^2+z^2}$. **6M**

OR

6 **a** Find the minimum value of $x^2+y^2+z^2$ given $x+y+z=3a$. **6M**

b If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$? **6M**

UNIT-IV

7 **a** Change the order of integration in $I=\int_0^1\int_{x^2}^{2-x}(xy)dydx$ and hence evaluate the same. **8M**

b Evaluate the following improper integrals $\int_1^\infty \frac{1}{x^4}dx$. **4M**

OR

8 Evaluate $\iiint z^2 dxdydz$ over the region defined by $z\geq 0, x^2+y^2+z^2\leq a^2$. **12 M**

UNIT-V

9 **a** Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx=\frac{1}{2}\,\beta(1,\frac{1}{2})$. **6M**

b Show that $\int_0^\infty x^4 e^{-x^2}dx=\frac{3\sqrt{\pi}}{8}$ **6M**

OR

10 **a** Show that $\int_0^{\pi/2}\sqrt{\sin\theta}d\theta.\int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}d\theta=\pi$. **6M**

b Prove that $\int_0^1(\log(1/y))^{n-1}dy=\Gamma(n), n>1$. **6M**

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